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Head and Flow Observations on a High-Efficiency Free Centrifugal-Pump Impeller

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A series of studies of the flow through the various components of hydrodynamic machinery is in progress in the Hydraulic Machinery Laboratory of the California Institute of Technology. Observations have been made on an impeller patterned after the Grand Coulee design. The impeller was operated as an isolated unit hydraulically free of the casing. The flow pattern at the discharge has been determined quantitatively for one flow rate, and a head-capacity curve for the impeller has been obtained. This paper constitutes a report on the findings up to the present.

Introduction

In the past most of the experimental work carried on in the field of rotating machinery has dealt with the machine as a whole, and because of the technical difficulties involved, comparatively little has been done to determine the performance characteristics of the individual elements which make up the whole. As a rough classification, such a machine can be thought of as consisting of three parts, namely, (*a*) the stationary inlet member, (*b*) the rotating member, and (*c*) the stationary outlet member.

Some empirical work has been carried out in which various rotating members have been tested with the same stationary members and vice versa, but the test results obtained in these cases have been referred to the performance of the combination as a complete machine, and the effect of changes in the individual member has been inferred only through the effect of such changes on the over-all performance. Several laboratories, particularly those of Spannhake, Thoma, and Pfleiderer, in Germany (1–9),[3] have undertaken experimental investigations of the detailed characteristics of the flow in the rotating passages, and a few workers have explored the flow during its transition from the rotating member to the case. In practically all instances, however, the experimental machine has been greatly simplified, usually to the point of making the runner two-dimensional. The gain from such simplification has been twofold, namely, the experimental difficulties have been lessened appreciably, and the possibility of parallel analytical studies has been improved. Unfortunately, the losses accompanying the simplification have included large decreases in efficiency, lowered resistance to cavitation, and a general lack of similarity to the performance characteristics of modern hydraulic machines.

Much effort has been devoted to the development of a satisfactory analytical treatment of the flow in hydraulic machines. Considerable progress has been made in the analysis of the axial-flow machine based upon airfoil theories, especially in the zone of efficient operation. However, for abnormal operating conditions even in the axial-flow machines, and for all conditions in the machines having appreciable components of radial flow, the presently known analytical methods leave much to be desired. If the performance of the individual elements of machines having good characteristics and high efficiencies could be obtained experimentally, and especially if the details of the flow could be determined, as well as the over-all characteristics of the elements, it would greatly enlarge the possibilities of developing a satisfactory analytical treatment and at the same time improve design possibilities through the use of more detailed empirical information.

In an effort to meet this need, a project has been initiated at the Hydrodynamics Laboratory of the California Institute of Technology under the sponsorship of the Office of Naval Research for the primary purpose of making detailed studies, both experimental and analytical, of flow through various components of well-designed modern hydraulic machines. Since the existing facilities of the Hydraulic Machinery Laboratory (10) were not readily adaptable to the type of experimental work anticipated, a smaller more suitable laboratory was constructed.

Laboratory Facilities

The project is located on the mezzanine floor of the Hydraulic Machinery Laboratory. The equipment is arranged to provide a flow of water in a closed circuit which is shown schematically in Fig. 1 and which consists essentially of three principal sections. One section functions primarily as a supply reservoir and includes the necessary equipment to deliver and meter a steady flow of water at various pressures and flow rates. It includes the reservoir and service pump, the Venturi meters and the throttle valve. This part of the circuit is independent of whatever test arrangement is made.

The second section functions principally as a distribution circuit and includes an adaptable arrangement of distributing headers, valves, and piping which may be arranged in various combinations to carry the flow from the throttle valve to and from the test elements in the particular manner and direction required by the unit under observation. Fig. 1 shows two of the several flow circuits possible. The major portion of this section is expendable and may be modified as the research requires.

The test stand is the third part of the circuit. Here are provided facilities for mounting, operating, and testing hydraulic-machine elements. The principal parts are the approach piping, the test basin, and the vertical dynamometer. These components are all in duplicate and thus allow two different studies to be run alternately and effect a great saving of time. Fig. 2 shows the test equipment as arranged for the photographic studies herein reported. One of the vertical dynamometers has been removed and replaced by a centerless impeller-drive mechanism to allow a full view of the impeller. Windows have been installed in the test basin to facilitate lighting and provide additional observation points. Fig. 3 presents the parallel test setup on the opposite side of the test stand. With this arrangement over-all

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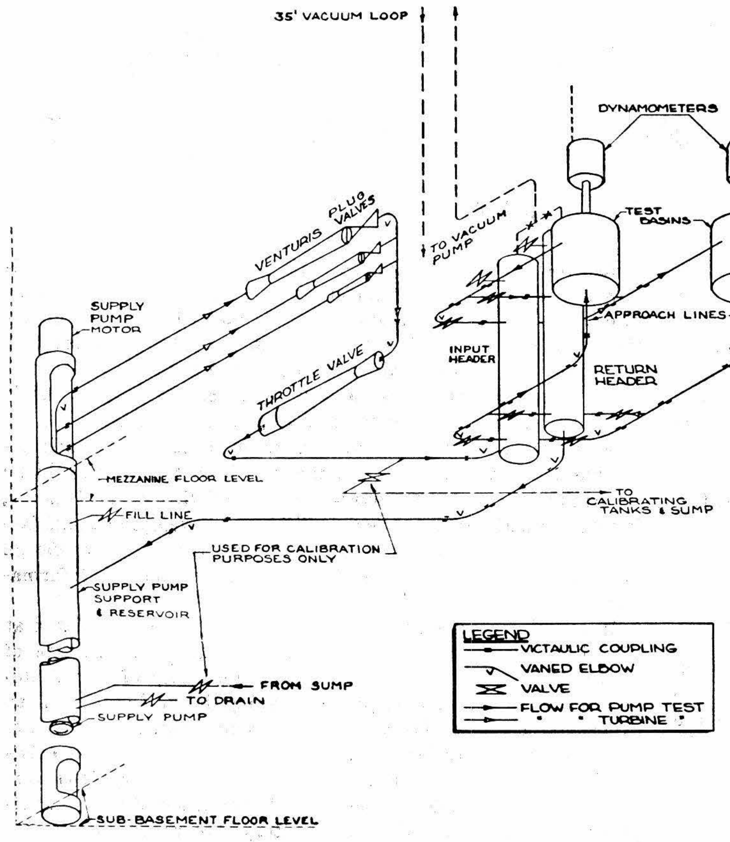

FIG. 1 DIAGRAM OF MAIN AND AUXILIARY FLOW CIRCUITS

head-capacity characteristics of the impeller were determined.

The project is a self-contained unit and can be operated independently of the other equipment in the building. However, to avoid duplication of equipment, the three Venturi meters may be connected to the main calibrating circuit of the Hydraulic Machinery Laboratory (10) and the existing facilities used for calibration. Fig. 2 shows the two points of interconnection.

The apparatus covers the following ranges:

1 Flow rates up to 4 cfs with a head differential of 66 ft at the test unit.

2 Power input or absorption up to 30 hp.

3 Dynamometers capable of rotative speeds of 100–2000 rpm in either direction.

The physical size of the test elements is not rigidly fixed. However, rotating channels up to 12 in. diam and diffuser or volute casings up to 30 in. diam may be accommodated.

Although the conventional opaque metal cast elements can be used in some test work, techniques have been developed at the laboratory for precision manufacture of these elements from transparent materials such as lucite. This not only makes possible the use of the various photographic techniques (12, 13) which have been developed and found to be extremely useful in observing and analyzing flow phenomena in the Hydrodynamics Laboratory (11), but it also offers advantages of controlled dimensions and guarantees symmetry and reproducibility to a degree essential for precise test work, which, ordinarily, are unattainable with standard cast elements.

Object of Tests

The impeller of a centrifugal pump was chosen as the first machine element to be examined. The rotating member was selected since it may be considered as the primary element of the pump; the principal interchange of energy between the machine and the fluid takes place in its rotating passages. The initial studies were confined to observations of the flow pattern at the discharge section of the impeller and to determinations of the over-all head-capacity characteristics of the impeller running free of its case. The discharge section of a modern medium-specific-speed impeller lends itself more readily to observation than does the inlet section and hence was selected as a logical starting point for a series of investigations which eventually will cover the entire impeller passage. Likewise, the head-capacity is the beginning of a series of studies designed to cover the complete over-all operating characteristics of the impeller. It is the purpose of this paper to report the findings of these initial studies and to present the tentative conclusions that have been drawn.

The Test Impeller

The test impeller used was very similar in design to the Grand Coulee pump impeller. This pump is a high-efficiency unit of medium specific speed, N_s = 100 (Q in cfs), and its design is representative of modern practice. The entire series of Grand Coulee model pumps was tested in the Hydraulic Machinery Laboratory and thus there is much detailed information available on the over-all performance of the pumping unit (15). Fig. 4 is a scale drawing of the impeller and presents its principal dimensions. The shroud curvature, the discharge and inlet vane angles, and the vane length were modeled to scale directly from the Grand Coulee design. However, the slight curvature or spooning of the vane in the eye was omitted and the vane was terminated along a straight line in the meridional projection. Although this alteration speeded production of the test impeller, it is not a limitation of the manufacturing technique; exact curvature of the prototype can be duplicated in the model if tests indicate the necessity. The discharge-vane tips were left blunt for the present tests.

A Free Impeller

It was required that the flow through the rotating impeller be independent of external effects. The flow was delivered to the impeller eye by a nozzle which matched the eye diameter exactly, and which was designed to give a uniform velocity profile at that section (14). The impeller was mounted directly above the nozzle and was rotated about a vertical axis. When using the centerless impeller mount, Fig. 2, the large journal bearing located between the impeller and the nozzle effects the necessary seal, whereas in the setup for the head-capacity measurements a ring of $^1/_4$-in-sq packing seated in a groove in the nozzle bears against the impeller and seals the suction side against leakage. It was intended originally that the impeller should be operated with a free atmospheric discharge, Fig. 5; however, as the capacity is reduced, thus lowering the static pressure in the eye, a point of instability is reached; the water breaks away from the top shroud and air flashes back into the impeller. This results in operating with passages partially filled, Fig. 6. Operating the impeller at extremely low speeds will avoid this difficulty, but the magnitude of the head generated and the flow rate are reduced correspondingly, and measurements become exceedingly difficult.

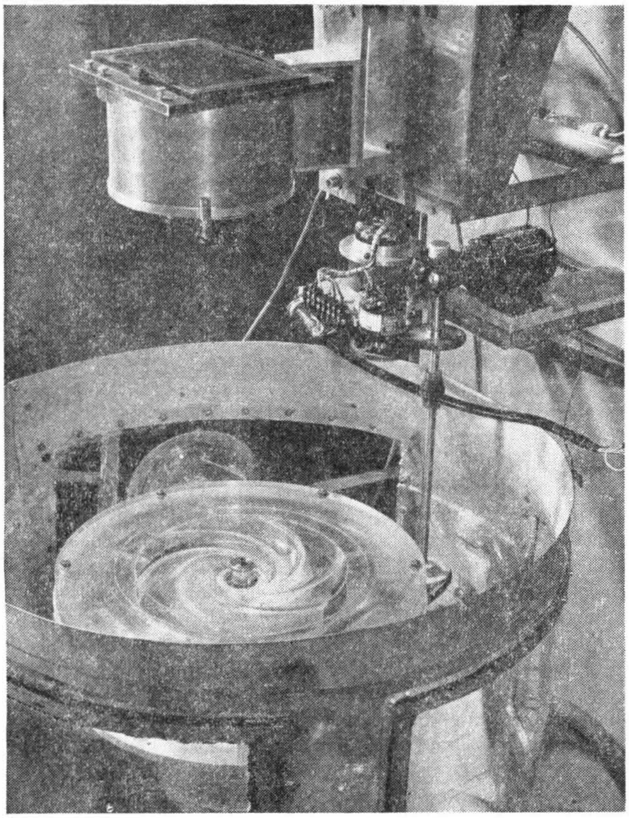

FIG. 2 EQUIPMENT FOR PHOTOGRAPHIC STUDIES OF FLOW

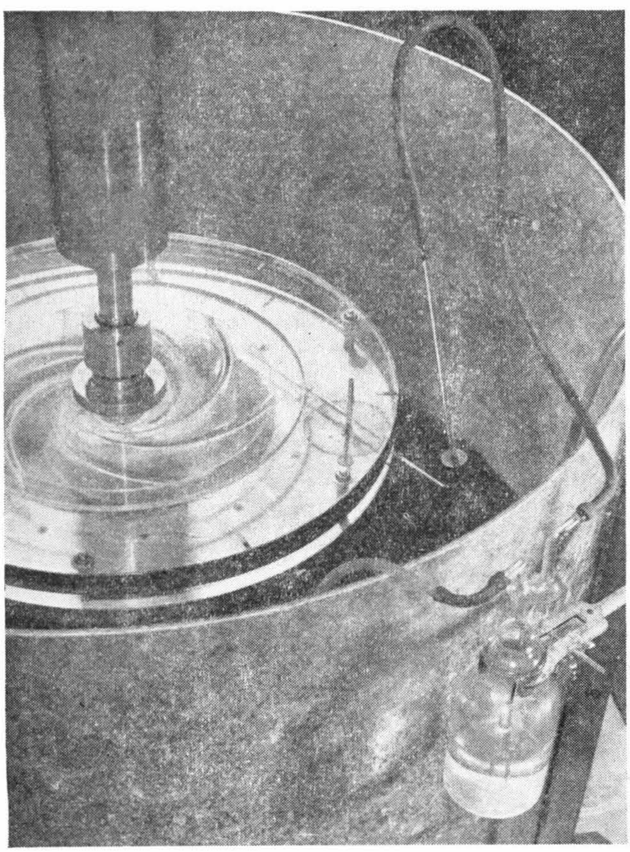

FIG. 3 EQUIPMENT FOR DETERMINING HEAD-CAPACITY CHARACTERISTICS

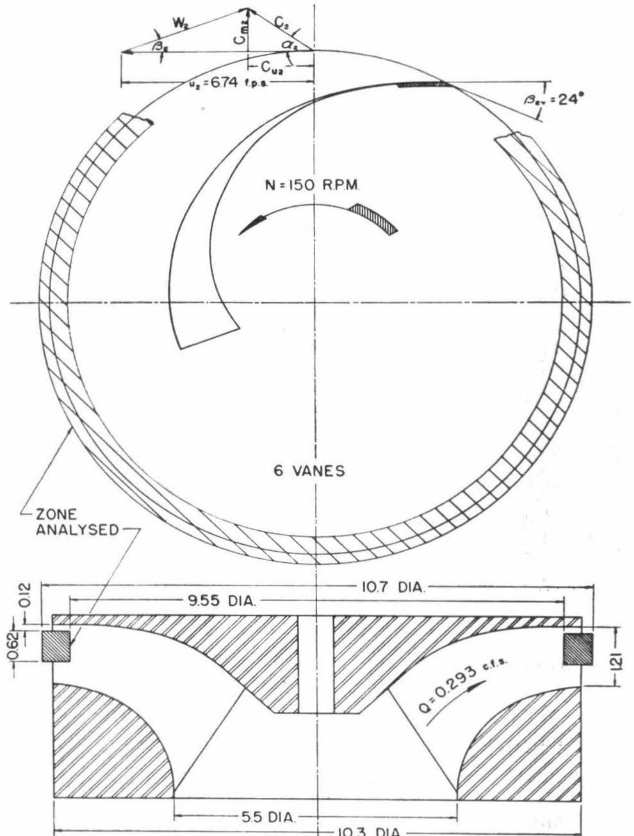

FIG. 4 PRINCIPAL DIMENSIONS OF TEST IMPELLER

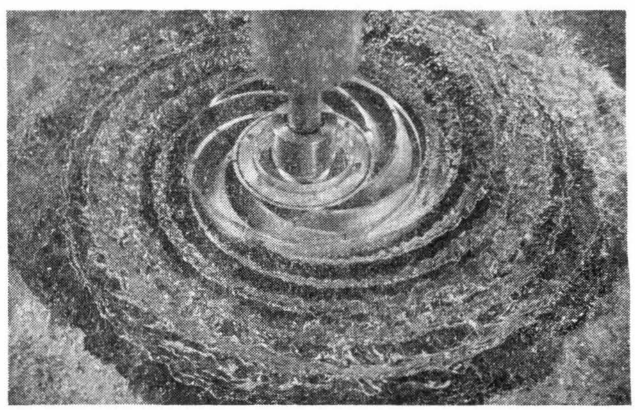

FIG. 5 AN IMPELLER DISCHARGING INTO A FREE BODY OF WATER AT HIGH CAPACITY; FULL PASSAGES

A symmetrical stationary collector about the discharge of the impeller furnished a satisfactory solution to the problem. The collector matches the impeller width at the periphery of the impeller and reduces to a smaller opening at its outer rim. A cylindrical weir is used to vary this opening and thus maintain a sufficiently high pressure in the collector and impeller to prevent flashback. The clearances between the impeller and the collector are close-running. Qualitative and quantitative studies of the flow pattern at the periphery of the impeller in the zone indicated in Fig. 4 were made with the impeller discharging freely, and also into the collector. These showed that the latter had no perceptible effect on the pattern of flow in that zone. With the exception of the flow in the boundary layer at the pe-

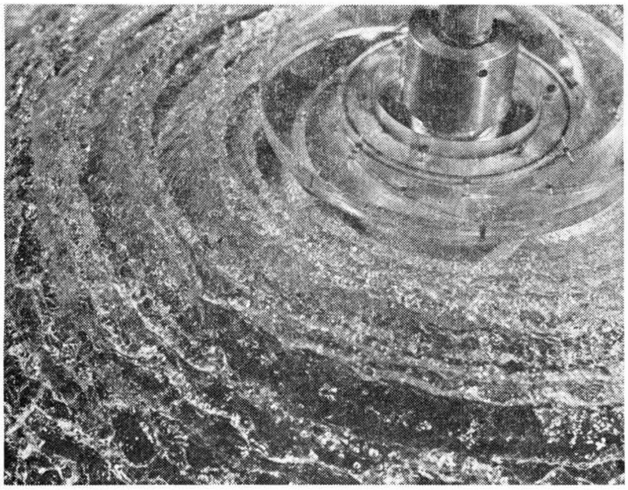

Fig. 6 An Impeller Discharging Into a Free Body of Water at Low Capacity; Partially Filled Passages

riphery, it is believed that the flow pattern at the discharge and throughout the entire impeller is unaffected. The test unit so arranged is considered to be hydraulically independent of the test setup and hence is referred to as a "free impeller."

Flow Observation

Instrumentation and Procedure. A three-dimensional photographic technique was used in making the quantitative flow studies. Thus the problems of instrument response and obstruction to the flow presented by mechanical methods were eliminated. However, photographic techniques required that the test passages be transparent and that suitable photographically identifiable particles be present in the stream. A mixture of dibutyl phthalate and kerosene proportioned to give a specific gravity equal to that of the water,[4] colored white and observed against a black background, was found to be satisfactory. This mixture is immiscible with water and, when injected into the flow, forms small globules which retain their identity and, when illuminated properly, are photographically discernible. Unlike the more common carbon tetrachloride-benzene solution, the mixture is not injurious to lucite. The tracers were released from a series of small capillary tubes arranged in the throat of the suction nozzle. Neglecting the minor surface-tension effect, the small tracers formed, $^1/_{32}$ to $^1/_{16}$ in. diam, may be considered to follow the same flow paths as the water itself.

[4] Equality was satisfactory when globules remained suspended in a sample of the water.

The tracer paths were recorded with a stereoscopic camera. The stereoscopic technique was necessary to establish the axial position or third dimension of the tracers in the passages. In Fig. 2 is shown the camera located above the periphery of the test impeller. A series of exposures were made on each plate with a controlled burst of flashes of a high-speed multiflash lamp. When the flash rate was matched properly with the impeller speed, the familiar golf-ball-type pictures resulted, Fig. 7. In this manner a number of positions of a tracer were recorded. To determine the tracer velocity, the stereophotographic images were projected back into space through the same lens system, that is, the camera was used as the projector and the space positions of the tracers at each interval of time were located. In this manner the angular, radial, and axial position of the center of each tracer group, the inclination of the group to the tangent, and the displacement of the tracers within the group were obtained. Knowing the time interval, the average velocity of each group was calculated. The shaded peripheral zone in Fig. 4 indicates the region covered by the analysis. The centers of all tracer groups fell within this zone.

The test impeller was operated at 150 rpm and a capacity of 0.293 cfs. At this capacity it was possible to operate the impeller both with and without the collector and hence make the comparative studies mentioned previously. The unit capacity for this operating point is 0.165, Fig. 11. Referred to the Grand Coulee prototype performance, this represents approximately a capacity 40 per cent above the maximum efficiency point and approximately coincides with the upper operating limit of that installation.

Discussion of Results. Figs. 8 and 9 present the results of the tracer studies. The plotted points were all obtained from tracer groups selected at random, the only requirement being that they be within the zone under observation.

The wide scatter of the data is most interesting. At first it might be thought to be experimental error. However, a variation of ± 50 per cent cannot be attributed to experimental errors which did not exceed ± 2 per cent. The second explanation that might be proposed is that the scatter was caused in part by the inclusion of data at various impeller radii and depths within the zone. No correlation between these factors has been observed and hence this contention is not supported by experimental evidence. The scatter could be attributed to any asymmetry existing in the flow through the impeller. However, tracer data were recorded in one specific channel and also in channels at random,

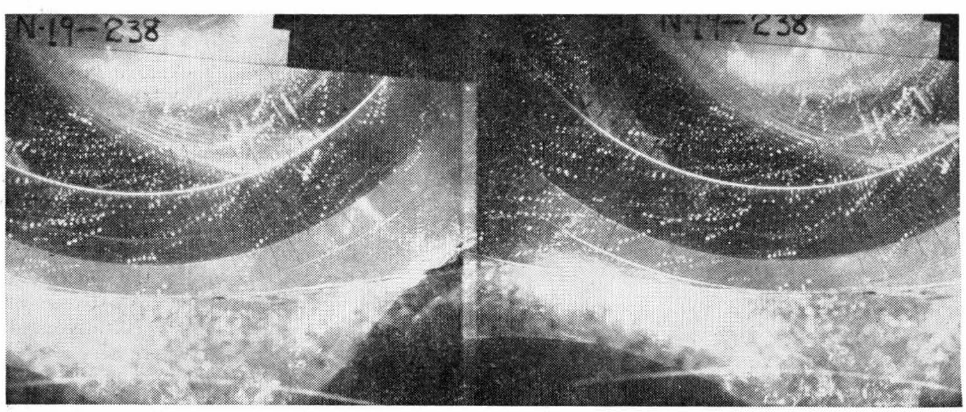

Fig. 7 Stereoscopic Photograph of Tracers in Impeller Passages

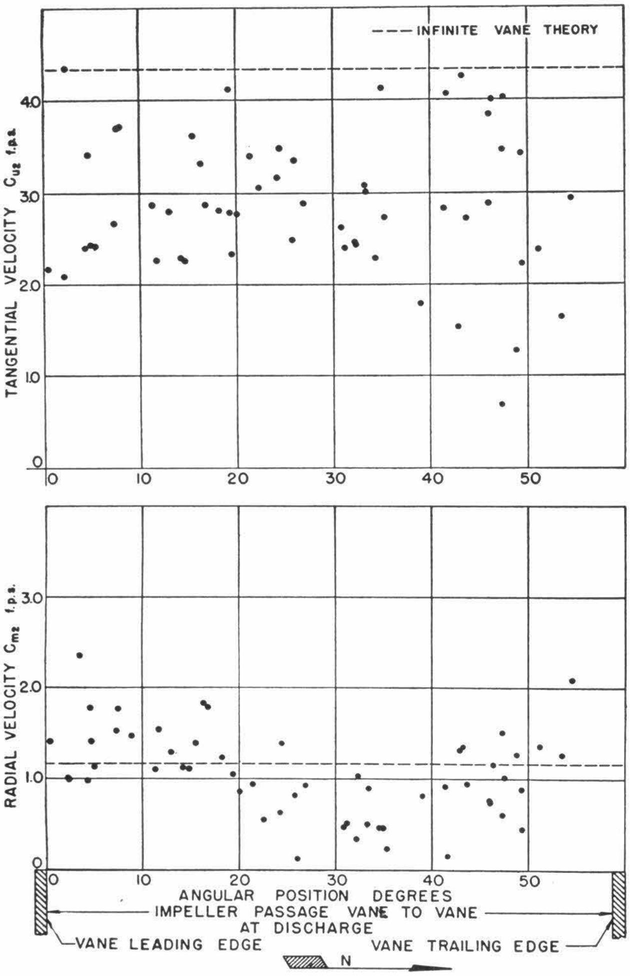

Fig. 8 Radial C_{m2} and Tangential C_{u2} Components of Absolute Velocity at Discharge

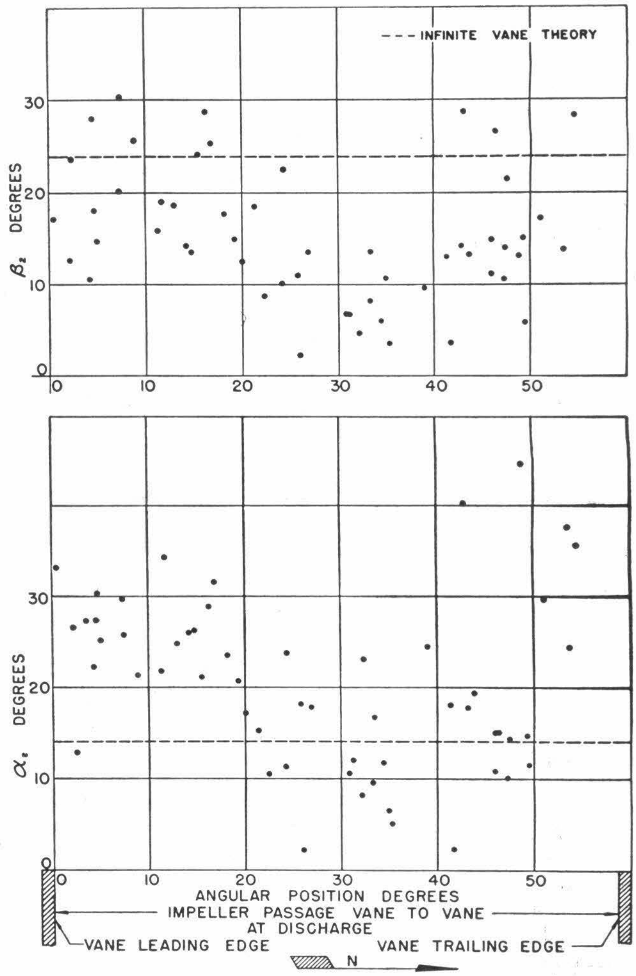

Fig. 9 Relative and Absolute Angles β_2 and α_2 at Discharge

and no asymmetry could be detected. It is possible that large-scale turbulence, present in the incoming flow, might be the source of this dispersion of data. Dye-streak studies have been made in the impeller approach over a wide range of capacities. Large-scale turbulence was not observed until the dye streaks came into close proximity with the impeller passages. It is concluded that the point scatter is due to large-scale turbulence which develops within the impeller passages.

It is interesting to note the increase in the scatter in the region near the trailing edge of the vane which strongly suggests an unstable flow condition. However, although an instability has been noticed in other photographic studies recently completed, a large rolling eddy has not been observed. The scarcity of points close to the trailing edge of the vane is a limitation imposed by the photographic method employed. When a series of exposures are made on one plate, the vane images obscure that area. Stereoscopic high-speed motion pictures overcome this difficulty.

The pronounced dips at mid-passage of the radial velocity C_{m2}, the relative discharge angle β_2, and the absolute discharge angle α_2 are the most striking features of the data. The decrease in β_2 is to be expected from theory. Likewise, theoretical considerations show that C_{m2}, β_2, and α_2 must each be of equal magnitude at opposite sides of the vane passage, and this point is verified. However, the C_{m2} profile is not yet substantiated by theory. Although a three-dimensional analytical solution of the problem is not available, the analysis of a two-dimensional straight radial-vane impeller indicates that, for that special case C_{m2} varies almost sinusoidally about a uniform average through-flow, being least near the leading edge of the vane and greatest near the trailing edge. The present data do not show this trend to exist in a modern three-dimensional impeller. The α_2 variation must necessarily follow the trend indicated, once C_{m2} and β_2 are established.

The obvious dip present in the other plots is noticeably absent in the C_{u2} data. The points suggest that the time-average specific energy about the periphery of an impeller is constant.

The horizontal dotted lines represent values of C_{m2}, C_{u2}, β_2, and α_2 based upon the Euler infinite-vane theory, namely, the relative discharge flow angle β_2 equals the vane angle β_{2v} and is everywhere constant, and C_{m2} is constant about the periphery. It is interesting to note that the position of this line with respect to the averages of the test data is in agreement with theoretical deductions. The Euler line must represent the average radial velocity, whereas it should lie above the mean β_2 and C_{u2} and below the mean α_2.

The instantaneous absolute streamlines pictured in Fig. 10 were sketched from a large number of data prints. The streamlines are spaced at random and hence represent flow direction

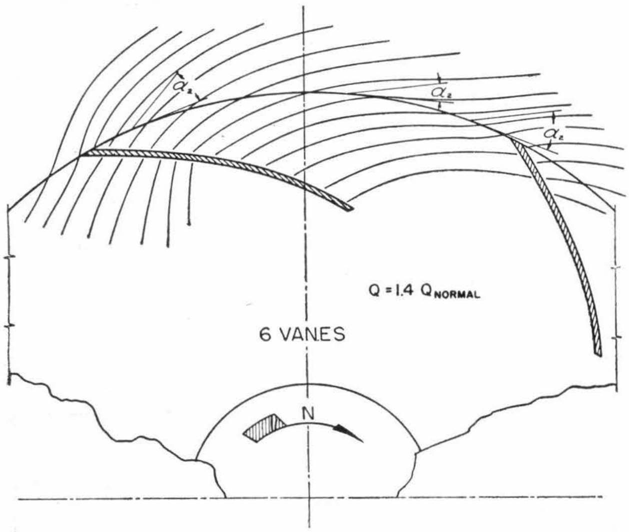

Fig. 10 Instantaneous Absolute Streamlines at Discharge; Random Spacing

only. It is interesting to note that in the region following the vane there is no evidence of backflow.

Determination of Head-Capacity Characteristics

Instrumentation. The inevitable losses involved in the use of a measuring point at any distance from the impeller made it desirable to use total head tubes close to the inlet and outlet to determine the energy of the inflow and outflow from the test impeller. Although there is some doubt as to the correct reference pressure at the inlet, considerations of symmetry indicate that the total head at some point on the center line of the suction nozzle seemed to be a desirable location at which to measure the total head of the incoming flow. A total-head tube directed upstream was located at the inlet to the pump impeller at a point $^7/_8$ in. below the plane of the intersections of the vane leading edges with the suction shroud. The total-head tube at the outlet was mounted on a rotatable mount and was retractible through a sleeve set at right angles to the axis of rotation. By these means the inlet to the total-head tube could be set at any predetermined distance from the impeller, and its axis could be aligned with the approaching flow. The direction of the flow at the nose of the total-head tube was determined by a probe carrying a fine thread.

Since it has not been possible to obtain a pressure gage with an accuracy and frequency response sufficient to follow in detail the variations in total head, a damped water-column manometer connected directly to the metal total-head tube was used. The air side of the manometer was connected to the air space of a waterpot. The water side of the waterpot was supplied by the total-head tube at the inlet to the impeller; thus the differential head across the runner could be read directly. By adjusting the height of the water level in the pot a large range in head could be covered without great variations in the height of the water level in the manometer tube, Fig. 3.

The question always arises in applications where the measured total head fluctuates, as to the meaning of the reading obtained by the manometer. Experiments were made to determine the influence of the damping imposed on the measuring head, and it was found that a large degree of damping gave steadier readings which were fully consistent with the mean readings obtained with less damping. Since the total-head tube measures a fluctuating energy level directly, and since speed and flow control were of such quality that the fluctuations could be considered a steady-state condition, a viscously damped system should read the true average total head.

Variation of Total Head Across Outlet. Since the flow through a radial-flow pump involves essentially a change of direction from axial to radial, it was of interest to determine the variation in total head between the lowermost streamline that emerges at the suction or lower shroud and the uppermost streamline that emerges at the back or upper shroud. This is particularly interesting in view of the velocity field observed photographically and discussed previously. A location was chosen for the total-head tube $^1/_{16}$ in. distant from the periphery of the impeller. At this location the total-head tube was aligned with the mean streamline and traversed vertically across the width of the impeller. At the chosen flow rates it was found that the energy of the lowermost streamline was approximately 5 per cent greater than the energy of the uppermost streamline. On the basis of these observations a location was chosen halfway between the two shrouds at the outlet for the subsequent work reported here.

Influence of Prerotation on Accuracy of Observations. Experimental work on complete pumps has shown that the flow conditions in the inlet pipe are in many cases highly complex. This requires that justification be provided for measuring the inlet total head with the total-head tube close to the impeller at the center line of the suction nozzle. This was done in the following manner:

The outlet total-head tube was set at $^1/_{16}$ in. from the periphery of the impeller at mid-width, and the total head change between the inlet and the outlet was recorded for a large range of flow rates. A typical run is shown in Fig. 11 which is a dimensionless head-capacity characteristic for the test impeller. Since it is reasonable to suppose that the inlet effects, if any, must decrease in intensity as one recedes further from the disturbing influence, other runs were made using the piezometer taps located after the inlet straightening vanes which are some distance upstream from the impeller inlet and guarantee a smooth velocity profile in the approach pipe.

The results of these measurements are plotted in Fig. 11, including the necessary corrections for approach-velocity head, but omitting the small loss due to the friction of the finite length of

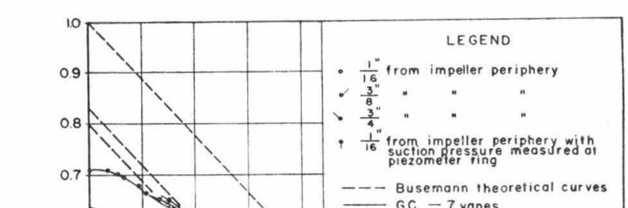

Fig. 11 Dimensionless Head-Capacity Characteristics of Test Impeller

pipe between the piezometer taps and the impeller eye. This latter correction has not been applied to the typical run shown in Fig. 11 and would improve the agreement at the higher values of dimensionless capacity where its influence is appreciable.

It should be observed that the points plot along the same line as the points determined using the total-head tube in the inlet. This is believed to indicate that in this installation the backflows which may exist do not affect the accuracy of the total-head tube in the eye. Since total head is the measure of the energy of the fluid at the point of measurement, it seems that there has been no energy supplied by the pump to the incoming fluid at the measuring point. Visual observations with dye introduced at the eye indicate that the disturbances in the eye are localized close to the vane leading edges.

Impeller Characteristics. The type of observation discussed in the previous paragraphs may be used to study some of the detailed qualities of an impeller. Near the point of zero head, the head-capacity curve shown on the unit plot in Fig. 11 is closely linear. In the range of maximum efficiency of the prototype impeller, the linearity has disappeared and the characteristic turns smoothly without discontinuities. At a value of $C_{m2}/u_2 = 0.075$, the smooth trend of the characteristic is broken, and in this region a scatter of points was obtained which indicate that the operation of the impeller is unstable. The region of instability is indicated by the dip in the characteristic. After this region is passed, unique values are again obtained for the dimensionless head.

As a matter of interest, the unit characteristic of the prototype impeller is plotted in Fig. 11 and is marked "GC-7 Vanes." The zone of discontinuity for this impeller is marked by two vertical lines which fall in practical coincidence with the zone of discontinuity of the six-vane test impeller. Work has not yet been completed on the five-vane impeller based on the same prototype to enable definite conclusions to be drawn from this coincidence. It indicates, however, that the position of the unstable range may depend primarily on the vane shape for impellers having sufficient vane overlap.

Prerotation. It was observed very early in the experimental work that the impeller developed zero head at a flow which still showed a positive tangential component of absolute velocity at the outlet. This has been verified very carefully both by observations of the absolute flow and by measurements with the total-head tube at the outlet. Since this condition requires that the flow enter the eye with a positive prerotation, there seems to be no doubt that such a phenomenon occurs, although visual evidence indicates that this is a local effect. The absolute angle of emergence of the flow at the periphery is quite acute and cannot be attributed to experimental error in the measurement of the direction of the outflow.

Variation of Characteristic With Position of Outlet Total-Head Measurement. The type of total-head tube used in these investigations was not sensitive to yaw of the flow over the angles encountered at the outlet of the impeller except, perhaps, in the neighborhood of the vane tips. The vane tips were left wide and unsharpened. When runs are made with the Pitot head further from the outlet than has been used in the previous experiments, lower total heads are measured. Fig. 11 shows a few points taken at $^3/_8$ in. from the periphery and $^3/_4$ in. from the periphery, and they define a characteristic lower than the one obtained with the measuring point $^1/_{16}$ in. from the periphery. On the basis of present evidence it is not possible to explain this shift. Variations in instrumentation have not caused any noticeable difference in this phenomenon, and it cannot reasonably be ascribed to losses due to mixing or to the losses in the rudimentary collector ring.

The lowest curve in Fig. 11 was obtained with the measuring point distant from the periphery of the impeller before the behavior of the total head near the impeller was noticed. This curve represents a minimum and undoubtedly contains losses due to the collector ring.

The head-capacity line, based on an infinite number of vanes and the outlet vane angle, is also shown in Fig. 11, as well as two lines for six and seven vanes, based on Busemann's (16) solution of the two-dimensional radial-flow impeller with a finite number of logarithmic spiral vanes. The displacement between the GC-7 vane characteristic and the Busemann line for seven vanes indicates that the characteristic for the test impeller is too high in relation to its corresponding Busemann line for six vanes. This indicates that the experimental data based on a measuring point very close to the outlet contain a systematic error. However, this discrepancy does not affect the experimental procedure when general characteristics of the runner are being investigated and efficiency determinations are not involved.

Influence of Collector Ring. The collector ring used in the determination of the characteristic consists of two parallel plates set apart the width of the outlet which were necessary to prevent air from striking back from the free surface. In order to throttle the outflow, one cylindrical ring was attached to the circumference of each collector-ring plate, and these could be set apart a determined amount. A systematic series of runs showed that in the range of operation there was no influence of the throttle gap on the measurements.

Affinity Relation. The data presented in Fig. 11 were obtained at a rotative speed of 225 rpm; at this speed the maximum head reading was 2.06 ft. This permitted a high accuracy of measurement of the differential head except near zero head. Flow measurements were accurate to better than 1 per cent at all flows.

With this accuracy available, another series of runs were made at 150 rpm to detect, if possible, the influence of rotative speed on the characteristic which could be significant at the low speeds used in these tests. The correspondence between the dimensionless head-capacity plots for the 225-rpm series and the 150-rpm series was complete within the accuracy of the experimental data.

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15 "Development of the Hydraulic Design for the Grand Coulee Pumps," by C. Blom, Trans. ASME, vol. 72, 1950, p. 53.

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Discussion

A. J. Acosta.[5] The need for a series of investigations on the individual elements of a hydraulic machine has been recognized for some time. Due to the somewhat inconclusive experimental results to date, it has been impossible to evaluate the separate effects of the components of a hydraulic machine. For that reason, the interpretation of observed phenomena and correlation with any real or perfect-fluid theory has lagged. Accordingly, the study of an impeller or volute, wherein its effects are isolated, should enable the laboratory to perform for the first time definitive tests as to the nature of flow in hydraulic machinery.

In reference to the plots of C_{m2} and C_{s2} (Fig. 8 of the paper), the authors state that there is as yet no theoretical justification for their behavior. The writer has had occasion to make an analysis of the flow of an inviscid incompressible fluid in a two-dimensional rotating-vane system. Inasmuch as an "ideal" solution offers possibilities of interpreting test data, it seems pertinent to present it now. The geometry of the chosen idealized impeller consists of eight logarithmic spiral vanes with a characteristic angle of 45 deg and a radius ratio of 0.5. The analysis involves the numerical evaluation of the differential equation governing the flow subject to appropriate boundary conditions. In the case considered, an absolute co-ordinate system (stationary with respect to the inertial reference frame) was chosen so that the differential equation to be solved is Laplace's equation in two dimensions, i.e., $\partial^2 U/\partial x^2 + \partial^2 U/\partial y^2 = 0$ where U is either the stream function or velocity potential for the flow. The boundary conditions are: (*a*) there is no flow through the vanes; (*b*) there is no flow around the tip of the vanes at exit. This latter condition is merely a statement of the Kutta-Joukowsky principle for the flow around lifting surfaces. The vanes are taken to be infinitely thin, although this condition is not necessary for the solution of the problem. Finally, we require that the velocity diminish to zero at an infinitely great distance from the impeller. Then, to within an additive constant, the stream function and velocity potential for the flow are uniquely defined, and the velocities in the field may be computed from either. The flow rate through the impeller is accounted for by placing a line source at the origin. Rather than compute a number of solutions for separate flow rates only two need be found, since the resulting velocity field within the impeller can be resolved into two parts, namely, (*a*) the flow due to the rotation of the vanes without any net through-flow (corresponding to shutoff head); (*b*) the flow from a source through a set of radially disposed stationary guide vanes. Solutions of this latter sort are available.[6] Combined with the present work which is only for zero flow rate, the flow for any operating point on the head flow-rate diagram may be obtained by taking a linear combination of (*a*) and (*b*).

The solution itself was effected by the so-called "relaxation" method.[7] Briefly, the method for solving Laplace's equation is as follows: The value of the function in the region of interest is approximately determined at the intersecting points of a rectangular grid. Then, to the first order, the finite-difference equation representing Laplace's equation is $\psi_1 + \psi_2 + \psi_3 + \psi_4 - 4\Delta\psi_0 = 0$, where ψ_1 to ψ_4 are the values of the function at the nearest four intersecting points to the point 0. This equation is satisfied at each point in the region by a trial-and-error process. Ways in which this is readily accomplished are discussed by Southwell.[7]

The solution was carried out by the method described, and from it the variation of the radial velocity across the impeller-exit section was determined and is shown in the accompanying diagram, Fig. 12. The value of the unit head at zero flow rate is 0.79, which is the same as that given by Busemann's theory[8] for logarithmic vanes. Although it is unfortunate that the dimensions of the impeller chosen for analysis are somewhat different from those of the test impeller, the order of magnitude of the velocities and the general features of the flow will be the same.

Comparing Fig. 8 of the paper with Fig. 12 computed by the writer, one sees that the dip in the radial-velocity distribution is substantiated by theory. For logarithmic vanes of the angle and radius ratio used in the analysis and for the test impeller, the radial-velocity variation at the impeller exit is nearly unaffected by the flow rate; so we are justified in comparing the variation at zero flow rate with that of the experiment. The range of variation seen in Fig. 8 is about a radial velocity of 1 fps, or in a dimensionless form it is 0.15, arrived at by dividing

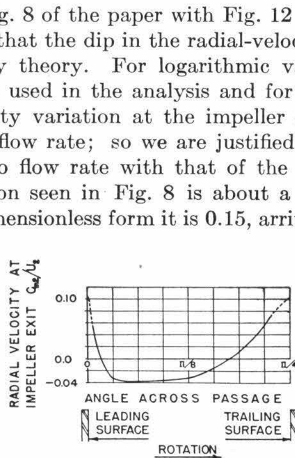

Fig. 12 Plot of Ratio of Radial Velocity to Peripheral Velocity at Exit Section Computed for a Two-Dimensional Impeller With Eight Logarithmic Spiral Blades Having a Vane Angle of 45 Deg and a Radius Ratio of 0.5 for Zero Flow Rate

by the peripheral velocity. The maximum variation computed from the two-dimensional ideal fluid case is about $\Delta C_{m2}/U_2 = 0.14$. The only noticeable departure in general trend between the two is found near the leading surface, where it is seen that the area of positive C_{m2} is much less for the computed case than for the test results. This is, however, the trend one would expect as the vane angle becomes steeper.

As the authors mentioned, the dip apparent in the C_{m2}, β_2, and

[5] Research Fellow in Hydrodynamics, California Institute of Technology, Pasadena, Calif. Jun. ASME.

[6] "Diagrams for Calculation of Airfoil Lattices," by A. Betz, NACA, T.M. 1022.

[7] "Relaxation Methods in Theoretical Physics," by R. V. Southwell, Oxford University Press, London, England, 1946.

[8] Refer to author's Bibliography (16).

α_2 is not evident in the C_{u2} (absolute tangential velocity at exit) plot. For the special case analyzed, the range of variation of C_{u2} compared with the mean value was found to be 18 per cent, 12.5 per cent being positive occurring at the boundary on the vane tip. It is, then, not surprising that a variation of this order of magnitude should not be observed on the authors' plot, in view of the large point scatter.

The general trend of the data presented is seen to follow that predicted by two-dimensional perfect-fluid theory, a surprising fact considering the high level of turbulence present in the flow. However, the limitations of perfect-fluid theory for flows of this sort are as yet unknown. The writer is interested in this problem and hopes to see it explored further.

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Authors' Closure

The authors thank Mr. Acosta for his discussion and consider it a valuable contribution. More complete and detailed studies made on a series of impellers since the completion of the paper indicate that in the region of the design point, the total head generated by the impeller matches rather well (within 8 per cent) the ideal Busemann value, and also that the head-capacity curve of the free impeller follows very closely the ideal straight-line relationship. In this region prerotation is nil; there is no separation within the passages and the discharge flow pattern is primarily two-dimensional. Hence, in this region, the two principal parameters present in the real fluid case but omitted in the ideal case are viscosity, μ, and the temporal velocity variations $\frac{\partial v}{\partial t}$. If through continued research the effects of these quantities on the ideal solution can be determined, an extremely powerful tool would be made available to the researcher and designer.

Unfortunately, the whole picture is not quite as simple as might be inferred. At off-design points, observations show that the flow separates from the channel walls and follows unknown and indeterminate boundaries. Cross flows develop and the discharge flow pattern does not remain two-dimensional. These elements impose serious limitations on the potential approach in the very regions where its use to predict off-design point performance would be most desirable. On top of all this, it must be remembered that the ideal situation applies only to a free impeller; an impeller operating in an infinite fluid medium with stationary boundaries only at infinity. Tests have shown that the characteristics of the complete pump, consisting of impeller and case, differ widely from those of the free impeller at all points except the design point. Hence the effect of the volute must be considered and incorporated in the final solution.

It is not meant to imply by these remarks that the potential solution is considered either impractical or impossible. The intent of the authors is to point out the long road ahead and the vast areas yet to be explored, and to underline the statement that at these early stages of the work it is most gratifying to find a good correlation between potential theory and experiment in some parts of the field.